A Study of Accretion Discs Using Computational Methods

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In this report we present a numerical implementation of a fluctuating accretion disc model in order to produce synthetic light curves, their associated power spectrum density (PSD) plots, as well as recreate the linear rms-flux relationship as observed in some astrophysical systems. Beginning with a brief theoretical background we provide a foundational basis for accretion disc theory as well as observational motivations for their study. We describe our model then investigate changing its model parameters on our output graphs. We conclude by proposing some extensions to our toy model.

Keywords: Accretion Disc, Alpha-Prescription

1. INTRODUCTION

1.1. Theoretical Background

Accretion discs are gravitationally bound structures composed of gas, dust, or other matter in orbit around a central mass. Ubiquitous in astrophysics, these discs are known to naturally arise surrounding a variety of objects such as black holes, white dwarfs and protostars. The formation of accretion discs may be described via the use of some initially simple physics, however accretion disc theory quickly leads to more complicated results incorporating fluid dynamics, plasma physics and many phenomenological estimates and guesses known only to specialists.[1] Nevertheless, simplifications and assumptions can be made, which lead to some tangible results. The study of accretion discs has wider physical implications, allowing us to help test more complex questions about black hole physics, understanding the growth of structure in the universe, or even meaningfully constrain our knowledge of the fundamental properties of space and time.[2]

1.1.1. Angular Momentum

A particle with angular momentum in orbit around a central gravitating body will remain in orbit. Thus in order to move to a smaller radius, the particle in question must lose some of its angular momentum,[3] this may only occur if there is an external torque acting on the particle, which is absent in a closed system. However if we now consider a bulk collection of particles in orbit around a central mass forming a disc, we may allow some particles to have more or less of the angular momentum than others. This redistribution of angular momentum between the particles allows for the radial transport of angular momentum outwards in the disc, a key focus in accretion disc theory. Following the calculations as shown by J.E. Pringle[4] it can be shown that an initially thin ring of rotating gas around a central object will evolve as to spread the disc out, with a small amount of mass at large radius containing the bulk of the initial angular momentum as shown in FIG. 1.

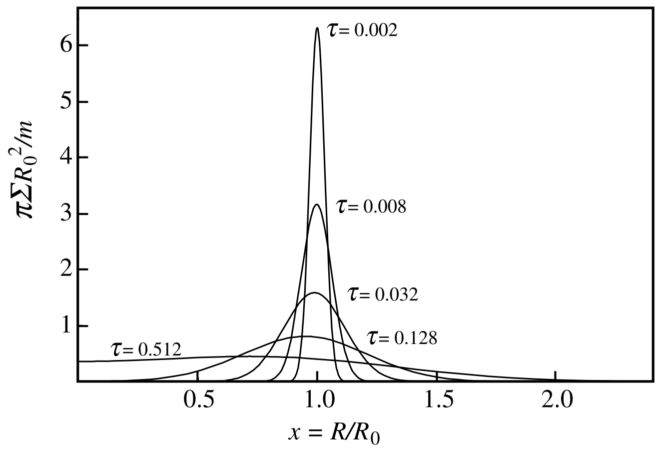

FIG. 1. Shows the viscous evolution of a ring of matter of mass m at radius $R = R_0$ in terms of dimensionless time τ position x and surface density Σ. One can see how the initial ring spreads, bringing the majority of the mass closer in while a small amount of the mass moves further out carrying with it the most of the angular momentum.[5]

This derivation relies on a parameter known as viscosity, which is the tool that converts gravitational potential energy into radiation in an efficient manner, it is an important feature in accretion disc theory and distinct from ordinary molecular viscosity. The exact process by which the angular momentum is redistributed has been of debate for some time. It is however suspected that a combination of several processes such as collisions between particles within the orbiting gas and possibly magnetorotational instability (MRI) are the likely perpetrators of angular momentum transport. As particles collide, shock heating, followed by radiative cooling acts to reduce the energy of the particles in the collision allowing for the redistribution of angular momentum within the system, similarly MRI is a fluid instability that causes chaotic changes in pressure and flow velocity within the disc.[6]

The process of angular momentum transport is not

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unique to accretion discs as it is not dissimilar to the process that occurs in other spiral structure such as galaxies[7]

1.1.2. Thermalisation and Emission

The process by which an accretion disc reaches thermal equilibrium is of particular interest as it is the process that gives rise to radiation and allows for any observation of the disc. The first and most straightforward process that a disc can produce electromagnetic radiation is simply that the disc exists in thermal equilibrium and thus produces black-body radiation based on its temperature, in fact, this assumption proves to be incredibly powerful and and can lead to results that accurately reflect observation.[8] We will talk about this in section 2.1.2. Other processes that can lead to radiation emission are Compton and inverse Compton scattering as well as emission absorption by hydrogen and helium inside the gas.[9]

1.2. Accretion Disc Theory

The study of accretion discs has been a major area of research over the past $\sim$ 40 years, and with with over 9000 papers published in the field last year, it remains active and topical. Currently, much of what we know in the field was developed in a burst of activity in the early 1970s in a series of important seminal papers.[10] One of these papers of special interest was one produced by Shakura and Sunyaev in 1973[11] who proposed an semi-analytical model for an accretion disc where an inflow of matter $\dot{M}$ at the outer regions of the disc was responsible for much of the observed spectrum. Some of the key results of the paper are that the effective temperature of a steady disc varied radially as $T(R) \propto T^{-3/4}$ as well as a parameter for the transport of angular momentum via the use of a dimensionless quantity known as "viscosity" denoted by α.[12]

1.2.1. The Viscosity Parameter α

The α parameter was initially proposed in terms of the kinematic viscosity ν, in a relation given by EQ. 1, and was also defined to be constant throughout the disc.

$$\nu = \alpha c_s H \tag{1}$$

Where c_s is the sound speed, H the height of the disc, and α is a free parameter that can take values between 0, corresponding to no accretion and approximately 1, and thus is essentially a measure of the efficiency of the angular momentum transport within the disc.[13]

From the standard Shukura-Sunayeav model of accretion discs we may define several key relations. The first of these is known as the viscous, or radial drift velocity, $V_{visc}(r)$. Given by EQ. 2, it is a measure of the speed of a particular process to propagate at a given radius r from the center of the disc.

$$V_{visc}(r) = \frac{1}{2\pi}\left(\frac{H}{R}\right)^2 r^{-0.5}\alpha \tag{2}$$

From the viscous velocity, we may define the viscous timescale, $t_{visc}(r)$ EQ. 3, which is the amount of time required for a process to propagate from a given radius r to the centre of the disc.

$$t_{visc}(r) = \frac{r}{V_{visc}(r)} = \frac{1}{2\pi}\left(\frac{H}{R}\right)^2 r^{1.5}\alpha \tag{3}$$

The viscous frequency $f_{visc}(r)$ may be defined as the reciprocal of the viscous timescale and thus is given by EQ. 4

$$f_{visc}(r) = \frac{1}{t_{visc}(r)} = \frac{2\pi}{\alpha}\left(\frac{H}{R}\right)^{-2} r^{-1.5} \tag{4}$$

It is also useful for understanding to briefly mention both the dynamical timescale $t_{dyn} \sim 1/\Omega$ where Ω is the orbital angular velocity, as well as the thermal timescale $t_{th} \sim c_s^2/\nu\Omega^2$. The dynamical timescale is an estimate of the smallest amount of time a physical process may take within the disc. The analogously for the thermal and viscous timescales. These three timescales may be generally be related by EQ. 5

$$t_{dyn} \ll t_{th} \ll t_{visc} \tag{5}$$

1.2.2. Fluctuating Accretion Disc Model

Initially Shakura-Sunyaev proposed that the dimensionless viscocity parameter α was a constant, however many models[14] have now been put forward that imply that possibly alpha could vary with radius in the disc making alpha now a function of radius $\alpha(r)$. Of paticular note is Lyubarskii in 1997[15], who proposed a variation of the standard thin disc model whereby the viscosity parameter α was not constant throughout the disc, but instead fluctuated independently at different radii. The reasoning behind this was that X-ray sources were observed to have random stochastic variations in their flux, and PSDs that displayed what is known as flicker noise, also referred to as 1/f noise or pink noise. It was found that the spectra could be explained via α fluctuations that occurred on the order of the local viscous timescale. A numerical implementation of this fluctuating accretion disc model with several extensions[16] is what we will be modelling in our project.

1.3. Observational Motivations

So far, the furthest discs we have been able to resolve discs that are around a distance of 3000pc,[17] this means that most of the observational evidence for the existence of discs around AGN or black holes often comes in the form of a total flux measurement (photometry) as well as measurements of spectral features (spectroscopy). These manifest themselves as observed light curves which is a fundamental instrument of observational study in the field.

1.3.1. Light Curves

Light curves are constructed via the observation of total flux in a certain energy band over a given period of time. By probing different parts of the energy band over different timescales we are able to build a rough picture of the processes involved in an accretion disc, a key observation is that that variations in higher energy domains (X-ray) tend to have faster variability than those at lower energy bands. This may imply that the processes that are responsible for x-ray production occur at much faster speeds and by consequence at much smaller radii.

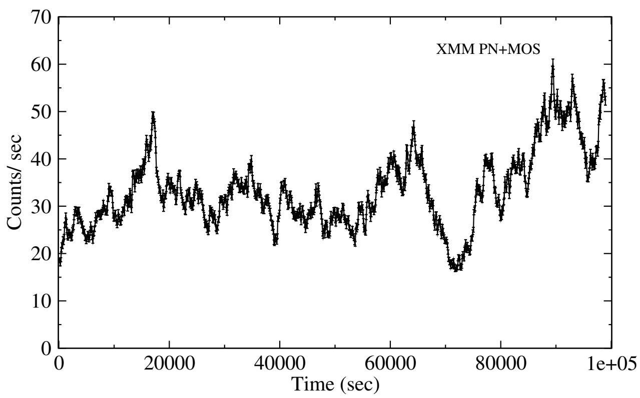

FIG. 2. An example of an actual observed light curve from the Narrow-Line Seyfert 1 Galaxy Ark 564 AGN.[18]

1.3.2. Power Spectrum Density

From a light curve, we may build what is known as a power spectral density (PSD or sometimes PDS), this is a method of analysis that provides information on the frequencies present within a time series. PSDs are incredibly useful as they can provide clues on various properties of the disc such as the nature of variability production and structure of a given accretion disc such as the position of inner and outer radii.

A PSD often displays a characteristic form and may be decomposed into some sort of function of frequency used to describe it. Often times this comes in the form of 1/f noise which represents a power spectrum with equal variability power per decade in frequency, while other times more complicated decompositions are required as shown in FIG. 3

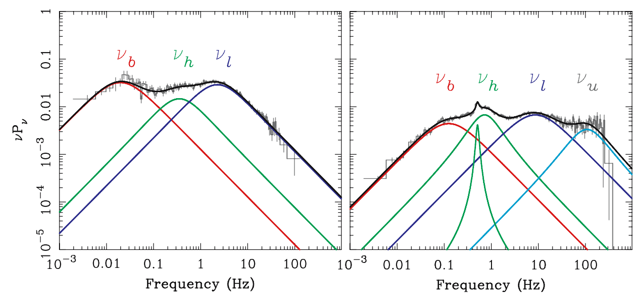

FIG. 3. Two PSDs from the hard state of GS 339-4 (left) and the neutron star 4U 0614+091 (right). One can see the associated decomposed Lorentzian components located at ν_b, ν_h, ν_l, as well as the additional Lorentzian ν_u present in the neutron star from additional noise power turbulence from the surface at the shortest timescales.[2]

1.3.3. RMS-Flux Relationship

The rms-flux relation relates the absolute root-mean-square (rms) variability of the light curve to its mean flux. It was observed that systems where accretion-induced variability is present that this relation often presents itself as linear.[19] It is a very good test to see if a given system has aperiodic variability on both short and long timescales that are coupled multiplicatively. The presence of a linear rms-flux relationship rules out shot-noise models, in which the different time-scales within the variability are combined additively.[20]

The rms-flux relation can potentially lead to the inference of constraints of certain parameters of the disc such as the value of the viscosity parameter α or the disc to height ratio H/R.[21]

To calculate the rms-flux relation, one must take a time series and divide it up into a discrete number of bins, and then by plotting the mean and rms of each bin against each other. These bins need not necessarily be equal in size, and in fact are often varied depending on the nature of the timescale of the variabiility.[22]

1.3.4. Outbursts, Hard/Soft States

X-ray binary systems (XRB) appear to display variability in their light curves over a large range of timescales. On the longest of these timescales, on the order of weeks to months, the PSD spectra of the system may resemble a hard power law and such is called the hard state. While on the shortest of timescales, the spectra may change dramatically as to resemble a quasi-thermal accretion disc in what is known as the soft state.[23] The transition from the hard to soft and back is refereed to as an "outburst" and can potentially be explained by the fluctuating disc model described briefly in section 1.3.5.

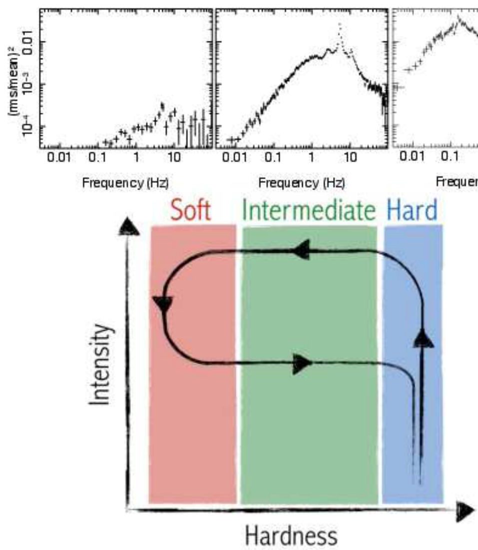

FIG. 4. Bottom: Schematic demonstrating the path traced on a hardness intensity diagram (HID) for a typical (idealised) black hole binary outburst. The source passes through three broadly defined states, the hard (blue), intermediate (green) and soft (red) states. Top: Power spectra representative of these three states, with the hard and soft states on the right and left respectively. From A. R. Ingram 2016[23]

1.3.5. Truncated Disc

The truncated disc model is an accretion disc model that is commonly used to explain spectro-temporal variability on both long and short time scales.In the truncated disk model, the inner areas of the disc are occupied by a hot gas that is thick in height ($H/R > 1$), and thus radiately inefficient, while the outer areas are occupied by a cooler geometrically thin gas that is radiately efficient.[24]

The structure of this report is organised as follows: In Section 2 we discuss our numerical model, mechanism of emission and main model parameters. In Section 3 we present our results of our model and discuss their significance. Section 4 contains our conclusions, possible extensions and closing remarks.

2. COMPUTATIONAL METHOD

2.1. Model Details

We have aimed to recreate the model put forward by P. Arevalo and P. Uttley,[25] which is a numerical implementation of the fluctuating accretion disc model.

Our disc model relies on a geometrically spaced (constant ratio) number of discrete annuli N, with an inner and outer radius r_{min} and r_{max}, the constant ratio between the annuli, c is given by EQ. 6

$$c = \left(\frac{r_{max}}{r_{min}}\right)^{1/N-1} \tag{6}$$

The choice of geometrically spaced annuli is in order to obtain what is known as equal power per radial decade which means that our resulting PSD will display the characteristic 1/f shape observed in some systems.[2]

Our disc is also assumed to have a height to radius ratio H/R as a variable parameter, as done by Arevalo and Uttley this value was normalised to 1 for simplicity. This means that our model is not actually one for a thin disc ($H/R \ll 1$), but instead lies in a sort of middle-zone between several different types disc models. The reason for this is that it is more computationally demanding to simulate thick discs than than thin (see FIG. 5) and thus explains why most numerical work in the last four decades have been on thick discs.[1]

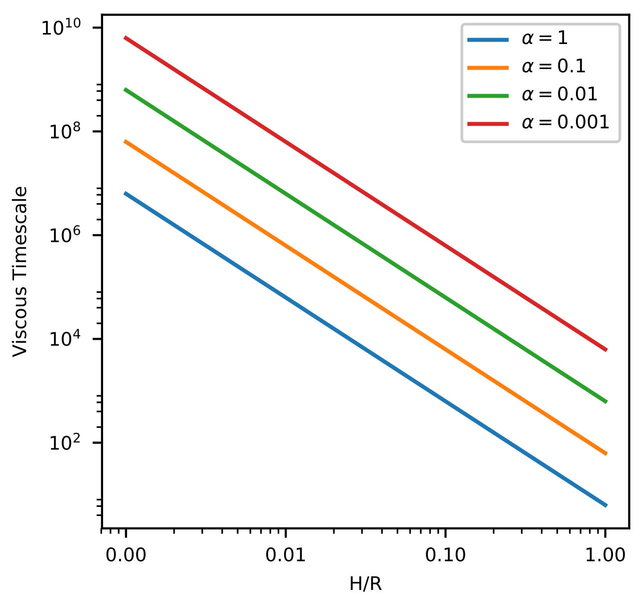

FIG. 5. Graph showing the relationship between the disc to height ratio H/R and the viscous timescale for several different values of α. We can see for small values of H/R and α the viscous timescale quickly becomes exponentially large and thus computationally unfeasable using normal methods.

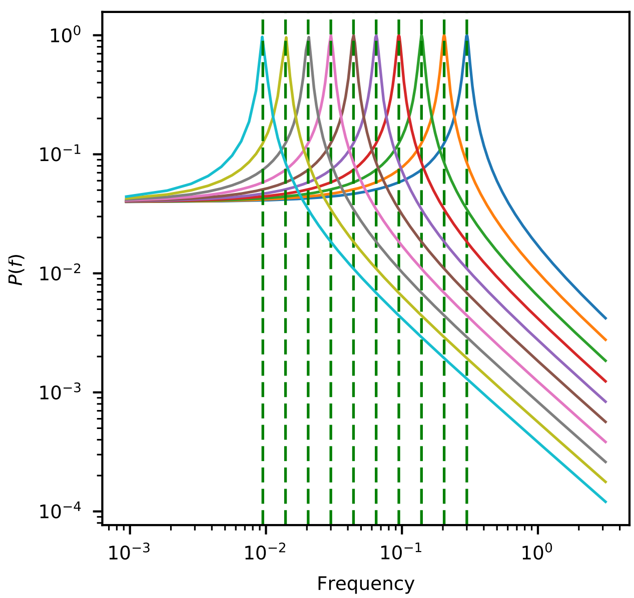

FIG. 6. Input Lorentzians with a quality factor of $Q = 0.5$, for the Timmer and Koenig method shown here for $N = 10$ annuli.

2.1.1. Timmer and Koenig Method

Each annulus is then assumed to have an independently fluctuating rate of mass accretion which we denote $\dot{m}(r,t)$ the generation of this time series is done via a process described by Timmer and Koenig[26] which allows for the generation of signal noise time series via the use of a power law spectrum. In other words we can create a time series with a desired PSD. For our model we used power law spectrum $P(\omega)$ that is was a Lorentzian shaped distribution cantered on the local viscous frequency of the particular annulus EQ. 7.

$$P(\omega) = \frac{\gamma^2}{\omega - \omega_{visc}^2 + \gamma^2} \tag{7}$$

where $\omega_{visc} = 2\pi f_{visc}$ and $\gamma = \frac{f_{visc}}{2Q}$ where Q is known as the quality factor which is the ratio of Lorentzian peak frequency to full width half maximum FHWM, and was varied between 0.5 and 10. A larger Q will create a broader Lorentzian at the same frequency.

Each small independent local mass accretion $\dot{m}(r,t)$ are also normalized so that they have a mean of 0, and a final fractional variability F_{var}. This small independent local mass accretion is essentially what accounts for the small α fluctuations described in section 1.2.2. Next we must introduce the propagation of these mass accretions through the disc, this is done via an iterative multiplicative process that propagates from the outermost to innermost radius as shown in EQ. 8

$$\dot{M}(r,t) = \dot{M}_0 \prod_{r_{max}}^{r_{min}} 1 + \dot{m}(r,t) \tag{8}$$

$\dot{M}_0$ is an initial arbitrary mass accretion rate at the outermost annulus that is then propagated inwards at every annulus and modulated by each of the radii further in by the factor of $1 + \dot{m}(r,t)$. In order to have the correct value in time, each $\dot{M}(r,t)$ must be shifted forwards in time by its local viscous timescale. This is to ensure that the propagation is actually causally connected in time and that the propagation did not actually occur instantaneously.

2.1.2. Emission Mechanism

To generate the light curve from our disc, we must define an emissivity profile which was done in two ways, the first is by applying physical concepts such as Plancks law in conjunction with the Stefan-Boltzmann law to obtain a spectral radiance as a function of radius of our disc. The second method involves simply assuming an emissivity profile loosely based on observation. In our model we will investigate both of these methods.

From EQ. 5.43 Accretion power in astrophysics[5] it can be shown that the dissipation rate $D(R)$ per unit area is given by EQ. 9

$$D(R) = \frac{3GM\dot{M}}{8\pi r^3}\left[1 - \left(\frac{r_{min}}{r}\right)^{1/2}\right] \tag{9}$$

Where G is the gravitational constant, M the mass of the central object, $\dot{M}$ the mass accretion rate, σ the Stefan-Boltzmann constant. This equation is one of the most commonly used equations in accretion disc theory and is essentially the flux of the disc.

Using the Stefan Boltzmann law $D = \sigma T^4$, we obtain an equation for the effective temperature EQ. 10

$$T(R) = \left(\frac{3GM\dot{M}}{8\pi r^3 \sigma}\left[1 - \left(\frac{r_{min}}{r}\right)^{1/2}\right]\right)^{1/4} \tag{10}$$

From this we can see if we omit the inner bracket ($R \gg R_*$) that the effective temperature goes as $T \propto r^{-3/4}$ as dictated by the Shukura-Sunayeav model.

From this effective temperature as a function of radius we may apply Plancks law as a function of frequency (EQ. 11 which provides the spectral radiance or also known as the specific intensity which provides the full radiometric description of a classical electromagnetic wave.

$$\epsilon(r) = I_\nu = B_\nu\left[T(r)\right] = \frac{2h\nu^3}{c^2(e^{h\nu/kT(r)} - 1)} \tag{11}$$

The units for spectral radiance are given by $erg \cdot s^{-1} \cdot cm^{-2} \cdot Hz^{-1} \cdot sr^{-1}$, and thus to obtain a resultant light curve we must specify a frequency ν and multiply the spectral radiance at every annulus by the particular annulis area which is given by EQ. 12

$$A = \pi\left(R_{i+1}^2 - R_i^2\right) \tag{12}$$

An alternative method for generating the spectrum of the disc was also used, as done by Arevalo and Uttley, one can simply assume an emissivity profile for the disc given by EQ. 13 then assuming that the emitted flux is simply proportional to the local mass accretion rate.

$$\epsilon(r) = r^{-\gamma}\left(1 - \sqrt{\frac{r_{min}}{r}}\right) \tag{13}$$

In this equation γ is a variable parameter that is in most cases $\gamma = 3$ as to follow the *total* energy loss. However certain energy bands may actually have radially dependent energy loss and thus γ is allowed to be variable for model robustness. This method provides similar results to the one derived from physical reasoning, however is computationally less intensive.

A plot of the emissivity vs radius for each method is shown in FIG. 7

2.1.3. Main Model variables

TABLE I. Tabulated are the main model variables, their associated symbols and typical values.

Name	Symbol	Typical Values
Number of annuli	N	$\approx 1000^{25}$
Minimum radius	r_{min}	$\approx 1.23 - 6R_g$
Maximum radius	r_{max}	$\approx 10^5 R_g$
Quality factor	Q	$0.5 - 10$
Disc Height Ratio	H/R	< 1 for thin discs > 1 for thick discs
Viscosity Parameter	α	$0.1 - 0.4^{13}$
Simulation time	t_{max}	N viscous timescales

The source code for our model may be found at `https://github.com/nx1/ModellingAccretionDisks`

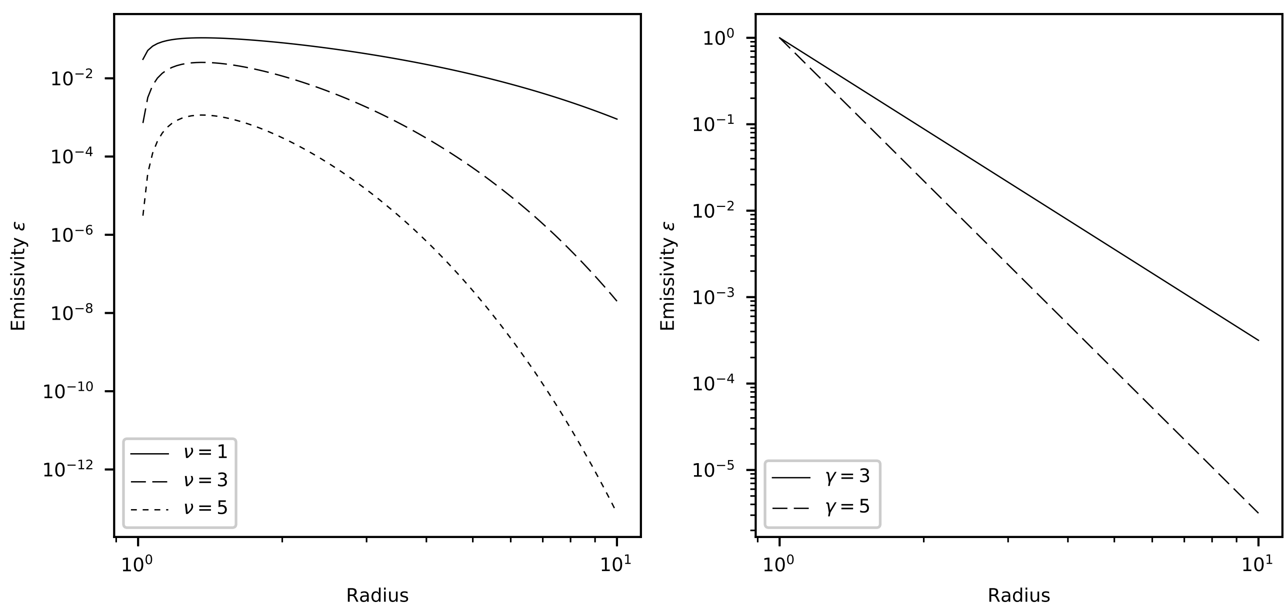

FIG. 7. The left figure shows the emissivity vs radius for the method in EQ. 11 with 3 different arbitrary frequencies ν shown. While the right shows the emissivity profiles from EQ. 13 for two values of γ. One can see that in both cases, the emissivity is stronger at radii further in, thus meaning the bulk of the emitted spectrum comes from the inner regions of the disc, this is especially true at higher frequencies.

3. RESULTS & DISCUSSION

Using the model described in section 2 we present several graphs and investigate the effect of changing model parameters on them. Units of length are given in gravitational radii $R_g = GM/c^2$. Units of time in R_g/c, and thus units of frequency in c/R_g.

3.1. $\dot{m}(r,t)$ Fluctuations as Sinusoids

In the initial stages of constructing our model, the small independent mass fluctuations $\dot{m}(r,t)$ described in section 2.1.1, was not modelled as a power noise time series, but instead modelled as simply being sinusoidal variations. This simplification is useful for testing and understanding the mechanism of mass propagation described in EQ. 8.

For the sinusoidal inputs for $\dot{m}(r,t)$ variations the following we used EQ. 14 where A is a scaling constant.

$$\dot{m}(r,t) = A \sin\left[2\pi f_{visc}(r)t\right] \tag{14}$$

Using $N = 3$ annuli, we can show the total mass accretion rate $\dot{M}(r,t)$ over a period of $1.1\times$ the maximum viscous timescale in FIG. 8.

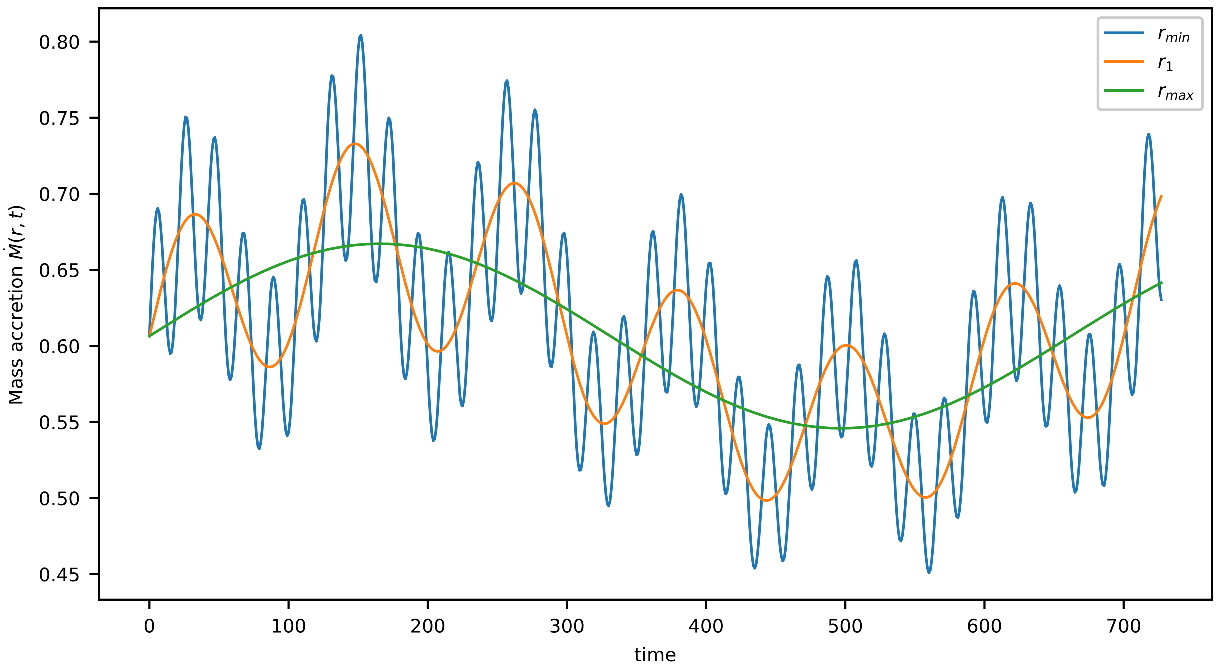

FIG. 8. Mass accretion rate $\dot{M}(r,t)$ against time with sinusoidal inputs for $\dot{m}(r,t)$. one can see how the initally long variabillity at the outer radii (green) modulates the variation at the next annuli (yellow) and then further modulates to the innermost annuli (blue). Variables for run: $N = 3$, $r_{min} = 1$, $r_{max} = 10$, $t_{max} = 1.1$, $H/R = 1$, $\alpha = 0.3$

We may also plot the rms-flux relationship for the above sinusoidal model and we indeed find an almost perfect linear relationship with an r^2 value of 0.9997 shown in FIG. 9, thereby confirming that the propagations are correctly propagated through the disc multiplicatively.

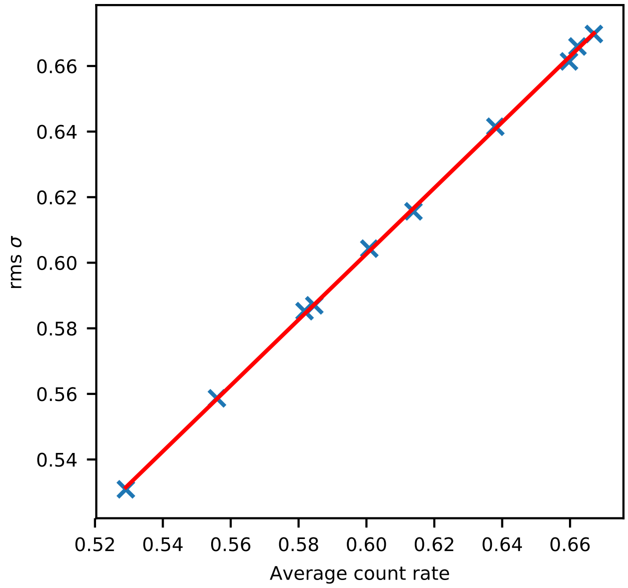

FIG. 9. The rms-flux relation for the mass accretion rate at the innermost radius $\dot{M}(r_{min})$ shown in FIG. 8. This relation was calculated over 10 bins of equal size in time.

3.2. Comparison with Arevalo and Uttley

FIG. 10 shows the results produced by the disc model put forward by Arevalo and Uttley in 2005.[25] One can see that the simulation is run over a period of $\sim 10^5$ units in time and the associated PSD displays a flat top characteristic of the typical 1/f a low frequencies then bends to steeper slopes at around $f_{visc}(r_{min}) = 3 \times 10^{-3} c/R_g$. The rms-flux relation displays the characteristic linear relation as expected of system with multiplicatively coupled time-scales.

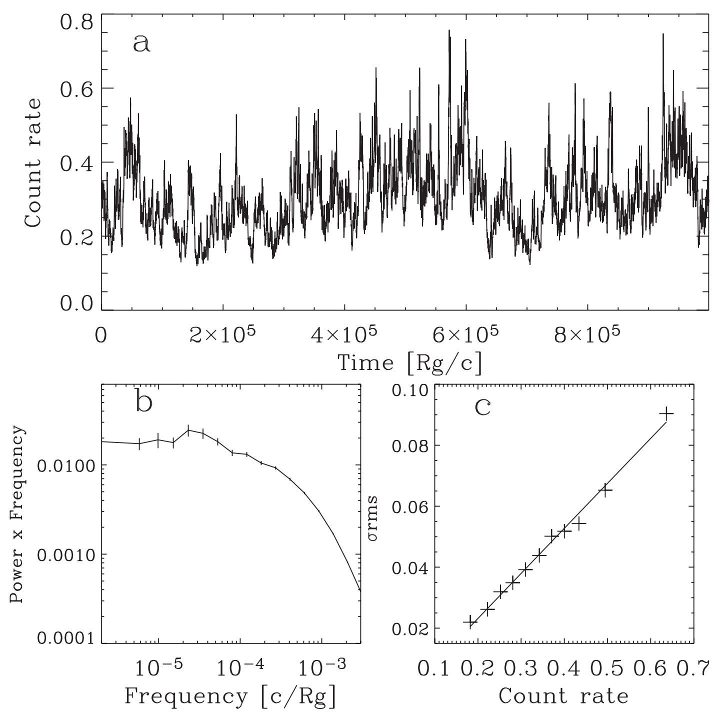

FIG. 10. Results from Arevalo and Uttley 2005[25] showing a realisation of a lightcurve (a), and associated PSD (b) and rms-flux relationship (c)

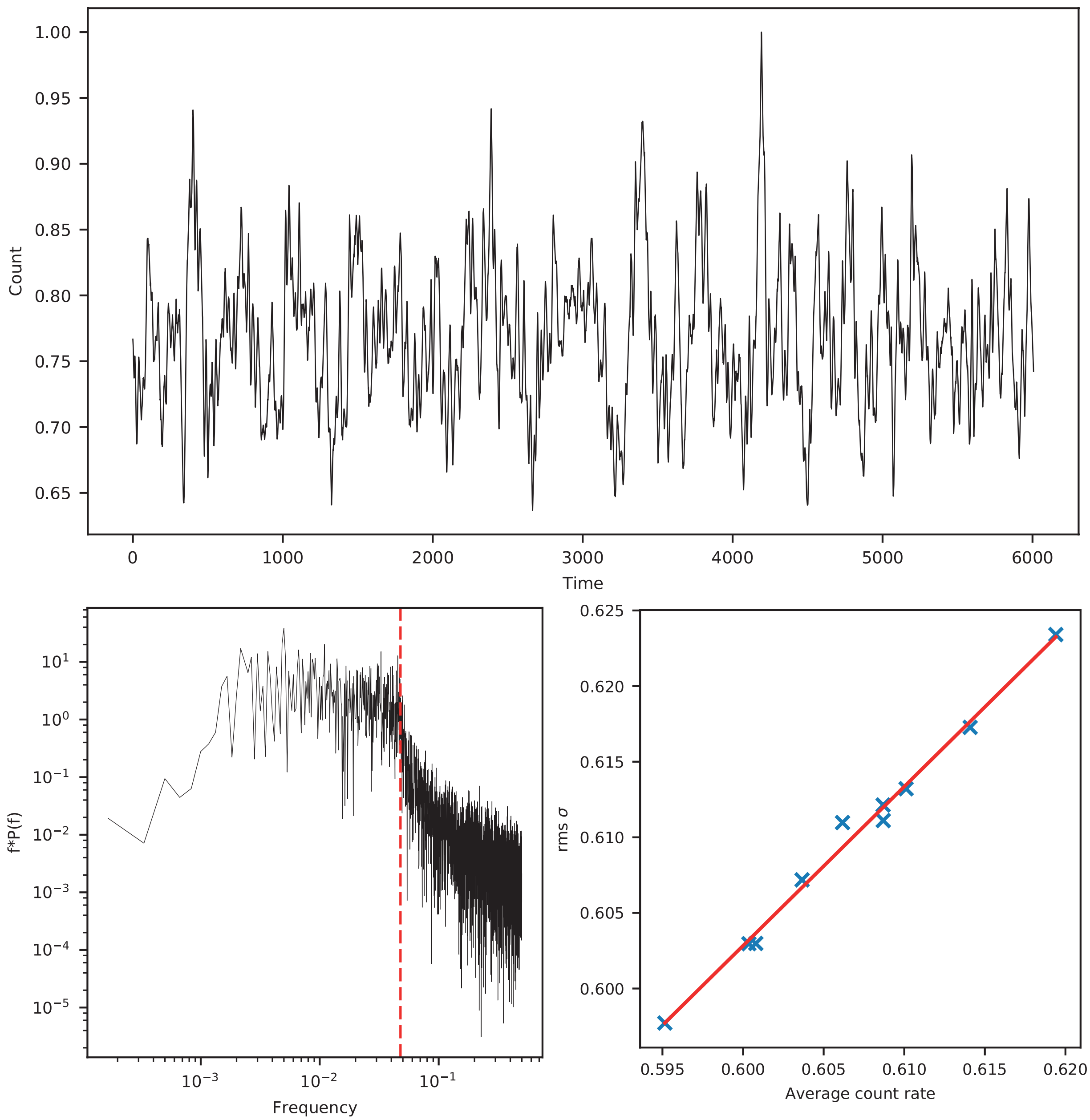

FIG. 11. Our results, attempting to replicate those in FIG. 10. Lightcurve for : $N = 100$, $r_{min} = 1$, $r_{max} = 10$, $Q = 0.5$, $t_{max} = 10.1$, $H/R = 1$, $\alpha = 0.3$. The red vertical line on the PSD indicates the maximum viscous frequency.

We present our version of these results in FIG. 11. There are some notable differences, the most obvious is the shape of the PSD the reason for which is due Areavalo and Uttley applying a low pass filter to their PSD via the use of a "filter factor", we have not implemented this, nevertheless one can see that our PSD still displays the characteristic flat top at lower frequencies and then a steep drop off at frequencies above the maximum viscous frequency $f_{visc}(r_{min})$ which has been labelled by a red vertical line. Our PSD also seems to display a break at lower frequencies that is not observed in the one put forward by Arevalo and Uttley.

Our light curve does seems to display a similar level of variability over the simulation time. Arevalo and Uttley have used $N = 1000$ annuli in their model and have likely used a much large maximum radius than we did for our model ($10R_g$), the reason for this is we experienced difficulty in creating PSDs for long time series and thus had to keep our model parameters such that our viscous timescales did not become too large.

3.3. Effect of Changing Radius

In this section we present a selection of light curves and PSDs for a combination of different radii (FIG. 12). The rest of the model parameters are kept identical to those in FIG. 11.

3.3.1. Changing Inner Radius

Increasing inner radii seems to acts as to reduce the the high frequency variability present in the light curve as well as shrink the size of the flat top area in the PSD by bringing reducing the value of the maximum viscous frequency. Therefore we can conclude that the location of the break is directly related to the position of the inner annuli, and postulate that the size of the flat top area on the PSD is related to the ratio of inner and outer radii.

3.3.2. Changing Outer Radius

Increasing the size of the outer radius has the effect of moving the break break on the left hand side of the flat area on the PSD to lower frequencies. This break is located at the lowest viscous frequencies associated with the outermost radii. The light curve responds to a larger outer radii by having a longer variability.

3.4. Effect of Changing Q

Changing Q to a larger value, corresponding to a broader input Lorentzian, acted as to reduce the sharpness of the PSD break. This parameter may prove useful when attempting fit a given model to observed PSDs as shown earlier in FIG. 3 for hard and soft states (FIG. 4). A possible model extension would be to have unique values of Q for each annuli.

3.5. Effect of Changing H/R

Attempting to reducing H/R served to increase the viscous timescale exponentially as described in section 2.1 and became increasingly computationally demanding. However our small amount of experimentation does not lead us to believe that changing H/R significantly affects either the light curve's general trend over N number of timescales, nor does it seem to significantly change the shape of the PSD in our model.

3.6. Effect of Changing α

Increasing the viscosity parameter α served to increase the viscous timescale at each radius and thus decrease the viscous frequency. However we do not notice much significant change in general shapes of the lightcurve, PSD or rms-flux relationship.

3.7. Effect of Changing Simulation Time

In our model, the simulation time of our model was set by the parameter t_{max} so that the total time over which the simulation took place was given by $Time = t_{max} \times f_{visc}(r_{min})$. We have chosen to simulate most of our light curves over around an arbitrary ~ 10 viscous timescales as it allows for the visualisation of good amount of variability. Setting t_{max} too high produces a light curve that appears completely noisey and random as any detail in the fluctuations are not sufficiently resolved. Using a t_{max} that is too low (< 2) causes an incomplete propagation of the fluctuations due to the time shifting described in section 2.1.1.

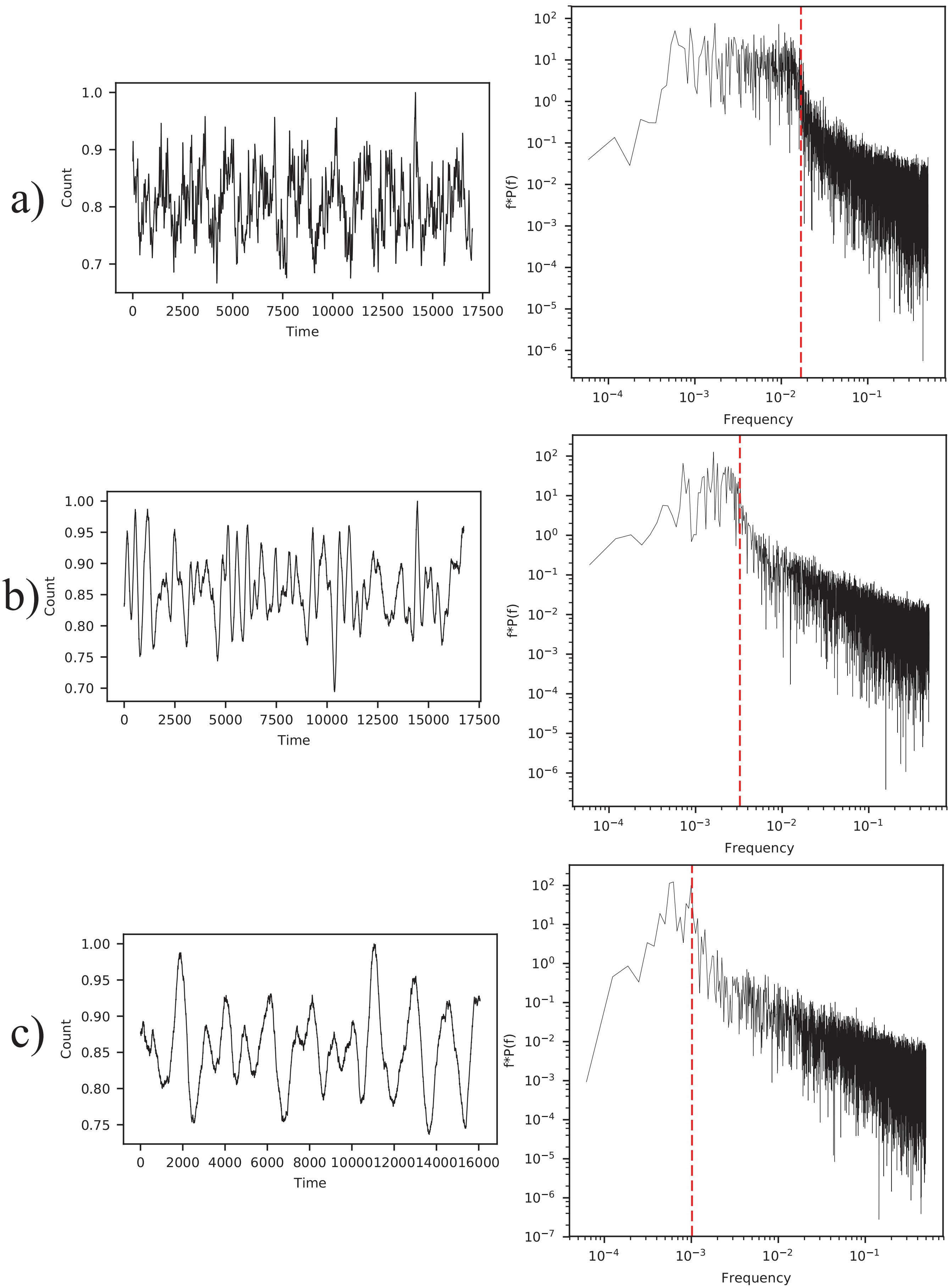

FIG. 12. Shows the effect of chainging the inner radius r_{min}, while keeping the outer radius fixed at $r_{max} = 20R_g$.
a) Corresponds to $r_{min} = 2R_g$ b) to $r_{min} = 6R_g$ and c) to $r_{min} = 13R_g$

4. CONCLUSION

Using a numerical implementation of a fluctuating accretion disc, we have been able to produce synthetic light curves, PSD and rms-flux graphs, and investigated the effect of varying model parameters on them. Visual comparisons have been made to results produced by Arevalo and Uttley, which seem to be congruent. We have successfully reproduced the typical 1/f shape with a break at the maximum viscous frequency seen in some system's PSDs, as well as the observed linear relationship between rms and mean flux.

We have also hopefully presented some useful background information regarding accretion discs that may help those aiming to build a foundation in the theory.

4.1. Extensions

Our model in its current state is a very primitive one and has significant room for improvements and additions that it would be impossible to mention them all. However one of the next logical steps in our model would be to attempt to create plots of the time lags of the outer and inner regions of the disc via the use of a cross correlation function.[27] Further extensions are not limited to attempting to produce a mechanism that accounts for the hard and soft states of the disc, attempting to fit the model to actual observed light curves, or extending the complexity of the model to include other emission processes.

Areas of investigation potentially lie within the emissivity profiles described in section 2.1.2, as well as the types of inputs in the Timmer and Koenig method.

More generally in accretion disc theory, addressing the problem of "turbulence" via the use of an arbitrary parameter α has not been satisfactory in the eyes of first principle physics. In the past, there have been attempts in the past to try and parameterise alpha in terms of some of the parameters of the disc, such as temperature, radius and density.[28] However a more commonly accepted proposition is that solutions may lie in the form of magnetorotational instability (MRI) that may be a generic source of turbulence in discs.[10],[6]

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